Numerical Methods With Matlab Programming

Subject code: 16SCCNM9

Unit - I

Matlab environment - getting started - Solving problems in matlab - saving you oaks - free define matlab functions - Using free define functions, manipulating matrices - computational limitation - special values and functions.

Unit - II

plotting : introduction two dimensional plots - three dimentional plotting - editing plotts from the menubar - creating plotts from the workshop window - programming in matlab : introduction - problems with two variables - input / functions - statement level control structures.

Unit - III

Numerical technics : Introduction - plot fitting - linear and polynomical regression - using the intractive fitting tools - numerical Integration - numerical differentiation.

Unit - IV

Low fitting - linear and ferabolic low by the method of least square

Principle - Solving algebric and transcedental equations - bisection method, half possition method and newton rapshon method - solving symethanias algebric equation - gauss seidal method - gauss elemination method.

Unit - V :-

Inter polation - Newtons forward and backward differents formulate - legrangious interpolation formulae - numerical integrations using trapiz and simpsons 1/3 rules - solution of ODE (ordinary Differential Equation) is equal to euler method - runge kuttak forcaller method.

Text books:-

M.K. vengatraman - Numerical methods Science and Engineering.

National publisher company 5th edition 2001.

M. Ellor, kuncicky, hollymoore - introduction to matlab.

③ Unit-IV

The solution of Numerical Algebraic and Transcedental Equation :-

Bisection or Balzano's or Interval halving Method :-

Problems:-

1. Find the positive roots of $x^3 - x = 1$. Correct to 4 decimal places by Bisection method.

Soln:

Let $f(x) = x^3 - x - 1$

Here $x = 1$, $f(1) = -1$ (-ve)

and $x = 2$, $f(2) = 5$ (+ve)

Hence a root lies between 1 and 2,

i.e., $x_0 = \frac{1+2}{2} = 1.5$

$f(1.5) = (1.5)^3 - (1.5) - 1$

$= 0.8750$

$x_1 = \frac{1+1.5}{2} = 1.2500$

$f(1.25) = -0.2969$

$x_2 = \frac{1.5+1.25}{2} = 1.375$

$f(1.375) = 0.2246$

$x_3 = \frac{1.25+1.375}{2} = 1.3125$

$f(1.3125) = (1.3125)^3 - (1.3125) - 1$

$= -0.0515$

$x_4 = \frac{1.3475 + 1.3125}{2} = 1.3438$

$f(1.3438) = (1.3438)^3 - 1.3438 - 1$

$= 0.0828$

$x_5 = \frac{1.3125 + 1.3438}{2} = 1.3282$

$f(1.3282) = (1.3282)^3 - (1.3282) - 1$

$= 0.0149$

$x_6 = \frac{1.3438 + 1.3282}{2} = 1.3360$

$f(1.3360) = (1.3360)^3 - (1.3360) - 1$

$= 0.0486$

$x_7 = \frac{1.3282 + 1.3360}{2} = 1.3321$

$f(1.3321) = (1.3321)^3 - (1.3321) - 1$

$= 0.0317$

$x_8 = \frac{1.3360 + 1.3321}{2} = 1.3341$

$f(1.3341) = (1.3341)^3 - 1.3341 - 1 = 0.0404$

$x_9 = \frac{1.3321 + 1.3341}{2} = 1.3331$

$f(1.3331) = (1.3331)^3 - 1.3331 - 1 = 0.0360$

$x_{10} = \frac{1.3341 + 1.3331}{2} = 1.3336$

$f(1.3336) = (1.3336)^3 - 1.3336 - 1 = 0.0882$

$x_{11} = \frac{1.3331 + 1.3336}{2} = 1.3334$

$f(1.3334) = (1.3334)^3 - 1.3334 - 1 = 0.0373$

$x_{12} = \frac{1.3336 + 1.3334}{2} = 1.3335$

$f(1.3335) = (1.3335)^3 - 1.3335 - 1 = 0.0878$

∴ The approx

$$x_{13} = \frac{1.3334 + 1.3335}{2} = 1.3335$$

$$f(1.3335) = (1.3335)^3 - 1.3335 - 1 = 0.0378$$

(5)

∴ The approachment root is 1.3335.

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2. Assuming that a root of $x^3 - 9x - 1 = 0$ lies in the interval (2, 4) find that root by Bisection Method?

Soln:-

Since the initial root x_0 lies between 2 and 4.

i.e., $x_0 = \frac{2+4}{2} = 3$

$$f(3) = 3^3 - 9(3) + 1 = -1$$

$$x_1 = \frac{4+3}{2} = 3.5$$

$$f(3.5) = (3.5)^3 - 9(3.5) + 1 = 12.3750$$

$$x_2 = \frac{3+3.5}{2} = 3.25$$

$$f(3.25) = (3.25)^3 - 9(3.25) + 1 = 6.0781$$

$$x_3 = \frac{3.5+3.25}{2} = 3.3750$$

$$f(3.8125) \; f(3.3750) = (3.3750)^3 - 9(3.3750) + 1$$
$$= 9.0684$$

$$x_4 = \frac{3.25+3.3750}{2} = 3.3125$$

$$f(3.3125) = (3.3125)^3 - 9(3.3125) + 1 = 7.5344$$

$$x_5 = \frac{3.3750+3.3125}{2} = 3.3438$$

$f(3.3438) = (3.3438)^3 - 9(3.3438) + 1 = 8.2928$

$x_6 = \frac{3.3125 + 3.3438}{2} = 3.3282$

$f(3.3282) = (3.3282)^3 - 9(3.3282) + 1 = 7.9124$

$x_7 = \frac{3.3438 + 3.3282}{2} = 3.3360$

$f(3.3360) = (3.3360)^3 - 9(3.3360) + 1 = 8.1020$

$x_8 = \frac{3.3282 + 3.3360}{2} = 3.3321$

$f(3.3321) = (3.3321)^3 - 9(3.3321) + 1 = 8.0070$

$x_9 = \frac{3.3360 + 3.3321}{2} = 3.3341$

$f(3.3341) = (3.3341)^3 - 9(3.3341) + 1 = 8.0557$

$x_{10} = \frac{3.3321 + 3.3341}{2} = 3.3331$

$f(3.3331) = (3.3331)^3 - 9(3.3331) + 1 = 8.0314$

$x_{11} = \frac{3.3341 + 3.3331}{2} = 3.3336$

$f(3.3336) = (3.3336)^3 - 9(3.3336) + 1 = 8.0435$

$x_{12} = \frac{3.3331 + 3.3336}{2} = 3.3334$

$f(3.3334) = (3.3334)^3 - 9(3.3334) + 1 = 8.0387$

$x_{13} = \frac{3.3336 + 3.3334}{2} = 3.3335$

$f(3.3335) = (3.3335)^3 - 9(3.3335) + 1 = 8.0411$

$x_{14} = \frac{3.3334 + 3.3335}{2} = 3.3335$

$f(3.3335) = (3.3335)^3 - 9(3.3335) + 1 = 8.0411$

∴ The approachment root is 3.3335.

3. Find the +ve root of $x - \cos x = 0$ by Bisection Method.

Soln:

Let $f(x) = x - \cos x$

$f(0) = -1$ (-ve)

$f(1) = 1 - \cos 1 = 0.4597$ (+ve)

Hence one roots lies between 0 and 1

$\therefore x_0 = \frac{0+1}{2} = 0.5$

Now,

$f(0.5) = (0.5) - \cos(0.5) = -0.3776$

$\therefore x_1 = \frac{1+0.5}{2} = 0.75$

$f(0.75) = (0.75) - \cos(0.75) = 0.0183$

$x_2 = \frac{0.5+0.75}{2} = 0.6250$

$f(0.6250) = (0.6250) - \cos(0.6250) = -0.1860$

$x_3 = \frac{0.75+0.6250}{2} = 0.6875$

$f(0.6875) = (0.6875) - \cos(0.6875) = -0.0853$

$x_4 = \frac{0.6250+0.6875}{2} = 0.6563$

$f(0.6563) = (0.6563) - \cos(0.6563) = -0.1360$

$x_5 = \frac{0.6875+0.6563}{2} = 0.6719$

$f(0.6719) = (0.6719) - \cos(0.6719) = -0.1107$

$$x_6 = \frac{0.6563 + 0.6719}{2} = 0.6641$$

$$f(0.6641) = (0.6641) - \cos(0.6641) = -0.1234$$

$$x_7 = \frac{0.6719 + 0.6641}{2} = 0.6680$$

$$f(0.6680) = (0.6680) - \cos(0.6680) = -0.1171$$

$$x_8 = \frac{0.6641 + 0.6680}{2} = 0.6661$$

$$f(0.6661) = (0.6661) - \cos(0.6661) = -0.1201$$

$$x_9 = \frac{0.6680 + 0.6661}{2} = 0.6671$$

$$f(0.6671) = (0.6671) - \cos(0.6671) = -0.1185$$

$$x_{10} = \frac{0.6661 + 0.6671}{2} = 0.6666$$

$$f(0.6666) = (0.6666) - \cos(0.6666) = -0.1193$$

$$x_{11} = \frac{0.6671 + 0.6666}{2} = 0.6669$$

$$f(0.6669) = (0.6669) - \cos(0.6669) = -0.1188$$

$$x_{12} = \frac{0.6666 + 0.6669}{2} = 0.6668$$

$$f(0.6668) = (0.6668) - \cos(0.6668) = -0.1190$$

$$x_{13} = \frac{0.6669 + 0.6668}{2} = 0.6669$$

$$f(0.6669) = (0.6669) - \cos(0.6669) = -0.1188$$

$$x_{14} = \frac{0.6668 + 0.6669}{2} = 0.6669$$

$$f(0.6669) = (0.6669) - \cos(0.6669) = -0.1188$$

∴ The approachment root is 0.6669.

Method of false Position:- (or) Regula Falsi:-

Consider the equation $f(x)=0$ and let $f(a)$ and $f(b)$ be of opposite sign. We have an approachment value for the root of $f(x)=0$, we get $\boxed{x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}}$

Note:-

order of the convergence of the Regula Falsi Method is 1.618.

Problem:-

1. Solve the +ve root of $x^3-4x+1=0$ by Regula Falsi Method.

Soln:-

$a < x_1 < b$

Let $f(x) = x^3-4x+1$

$f(0) = 1$ (+ve)

$f(1) = 1-4+1 = -2$ (-ve)

$\therefore$ The root lies between 0 and 1.

$0 < x_1 < 1$

Here $a=0$ and $b=1$

WKT:-

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{0(-2)-1(1)}{(-2)-1}$$

$$= \frac{-1}{-3} = 0.3333$$

$$\therefore f(x_1) = f(0.3333) = (0.3333)^3-4(0.3333)+1$$

$$= -0.2962$$

$0 < x_2 < 0.3333$

$$x_2 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

(10)

$$= \frac{0(-0.2962) - (0.3333)(1)}{-0.2962 - 1}$$

$$= \frac{-0.3333}{-1.2962} = 0.2571$$

$f(x_2) = f(0.2571) = -0.0114$ $\quad 0 < x_3 < 0.2571$

$$x_3 = \frac{0(-0.0114) - (0.2571)(1)}{-0.0114 - 1} = 0.2542$$

$f(0.2542) = -0.0004$ $\quad 0 < x_4 < 0.2542$

$$x_4 = \frac{0 \cdot (-0.2542) - (0.2542)(1)}{-0.2542 - 1} = 0.2541$$

$f(0.2541) = 0.0000$

∴ Hence a root is 0.2541.

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1. Find a positive roots of $xe^x = 2$ by the method of Flase position.

Soln:-

Let $f(x) = xe^x - 2$

$f(0) = -2$

$f(1) = 0.7183$

∴ A root lies between 0 and 1.

WKT:-

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} \qquad a < x_1 < b \quad 0 < x_1 < 1$$

$$= \frac{0(0.7183) - 1(-2)}{0.7183 - (-2)} = 0.7358$$

$f(x_1) = f(0.7358) = -0.4643$ $\quad 0.7358 < x_2 < 1$

$x_2 = \phi(0.7358) = (0.7358)(-0.4643) - 1$

(11)

$$x_2 = \frac{(0.7358)(0.7183) - 1(-0.4643)}{0.7183 + 0.4643}$$

$$= \frac{0.5285 + 0.4643}{0.7183 + 0.4643} = \frac{0.9928}{1.1826} = 0.8395$$

$f(x_2) = f(0.8395) = -0.0564$ $\quad 0.8395 < x_3 < 1$

$$x_3 = \frac{-(0.8395)(0.7183) - (1)(-0.0564)}{0.7183 - (-0.0564)}$$

$= 0.8512$ $\quad 0.8512 < x_4 < 1$

$f(x_3) = f(0.8512) = -0.0061$

$$x_4 = \frac{0.8512(0.7183) - (1)(-0.0061)}{0.7183 - (-0.0061)}$$

$$= \frac{0.6114 + 0.0061}{0.7183 + 0.0061} = \frac{0.6175}{0.7244} = 0.8524$$

$f(x_4) = f(0.8524) = -0.0009$ $\quad 0.8524 < x_5 < 1$

$$x_5 = \frac{0.8524(0.7183) - (1)(-0.0009)}{0.7183 - (-0.0009)}$$

$$= \frac{0.6123 + 0.0009}{0.7183 + 0.0009} = \frac{0.6132}{0.7192} = 0.8526$$

$f(x_5) = f(0.8526) = 0.0000$

$\therefore$ Hence a root is 0.8526.

H.W:

1. Solve the equation $x \tan x = -1$ by regula falsi method starting with $a = 2.5$ and $b = 3$ correct to three decimal places. (12)

Newton-Raphson or Newton's Method

We know that $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Convergence:

If $|\phi'(x)| < 1$, where $\phi(x) = x - \frac{f(x)}{f'(x)}$

Note 1:

This method is also called method of tangents.

Note 2:

The order of convergent by NR method is 2.

1. Find the +ve root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton Raphson method correct to 5 decimal places.

Soln:

Let $f(x) = 2x^3 - 3x - 6$, $f'(x) = 6x^2 - 3$

WKT:

Newton Raphson

$$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$$

Now $x_0 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{4}{21} = 1.80952$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= (1.80952) - \frac{0.42149}{16.64618} = 1.80952 - 0.02532$$

$$= 1.78420$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= (1.78420) - \frac{0.00694}{16.10022} = 1.78420 - 0.00043$$

$$= 1.78377$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= (1.78377) - \frac{0.00002}{16.09101} = 1.78377$$

∴ Hence a required root is 1.78377.

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2. Find the real +ve root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places.

0.607102

Soln:

Let $f(x) = 3x - \cos x - 1$, $f'(x) = 3 + \sin x$

Wkt:

Newton Raphson

$$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$$

Now $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{1.459698}{3.841471} = 1 - 0.379984 = 0.620016$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.620016 - \frac{0.046179}{3.581048} = 0.620016 - 0.012895$$

$$= 0.607121$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.607121 - \frac{0.000069}{3.570505} = 0.607121 - 0.000019$$

$$= 0.607102$$

∴ Hence a required root is 0.607102.

Solution of Symmetanious linear Algebric equation :

Direction Method :

Gauss Elemination Method :

1. Solve the system by Gauss Elemination Method

$2x+3y-z=5$
$4x+4y-3z=3$
$2x-3y+2z=2$

Soln:

Given equations : $2x+3y-z=5$
$4x+4y-3z=3$
$2x-3y+2z=2$

WKT: $AX=B$

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

Also,

$$(A,B) = \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{array}\right]$$

$$(A,B) \sim \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & -6 & 3 & -3 \end{array}\right] \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 6 & 18 \end{array}\right] R_3 \to R_3 - 3R_2$$

By using back substitution method,

$6z = 18$

ie., $z = 3$

$-2y - z = -7$

$-2y - 3 = -7$

$-2y = -4 \Rightarrow y = 2$

Also, $2x+3y-z=5$

$2x + 3(2) - 3 = 5$

$2x = 2$

$x = 1$ //.

2. Solve the system of equation by Gauss elimination method. (16)

$x+2y+z=3$
$2x+3y+3z=10$
$3x-y+2z=13$

The given equation $x+2y+z=3$
$2x+3y+3z=10$
$3x-y+2z=13$

WKT: $AX=B$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 13 \end{bmatrix}$$

Also,

$$(A,B) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

$$(A,B) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & -1 & 2 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] \quad R_3 \to R_3 + (-7)R_2$$

By using back substituting method we have

$-8z = -24$
$z = 3$

W.K.T $-y+z=4$
$-y+3=4$
$y=-1$

Also, $x+2y+z=3$
$x+2(-1)+3=3$
$x+1=3$
$x=2$

$\therefore x=2, y=-1, z=3.$

Gauss seidel :- (Iterative Method)

Sufficient condition:

$|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

1. Solve the following system by Gauss seidel method.

$10x - 5y + 2z = 3$

$4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

Soln:

Given equation $10x - 5y + 2z = 3$

$4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

$|10| > |-5| + |2|$

$|10| > |4| + |3|$

$|10| > |1| + |6|$

WKT: The diagonal elements are dominent (diagonally dominent)

$10x - 5y + 2z = 3$

$x = \frac{1}{10}[3 + 5y - 2z]$ → (1)

$4x - 10y + 3z = -3$

$y = \frac{1}{10}[3 + 4x + 3z]$ → (2)

$x + 6y + 10z = -3$

$z = \frac{-1}{10}[3 + x + 6y]$ → (3)

1st Iteration :-

$x = \frac{3}{10} = 0.3000$ $(\because y = 0, z = 0)$

$y = \frac{1}{10}[3 + 4(0.3) + 0]$ $(\because x = 0.3, z = 0)$

$y = 0.4200$

$z = \frac{-1}{10}[3+(0.3)+6(0.42)]$ $(\because x=0.3,\ y=0.42)$

$z = -0.5820$

2nd Iteration:-

$x = \frac{1}{10}[3+5(0.42)-2(-0.582)] = 0.626$

$y = \frac{1}{10}[3+4(0.626)+3(-0.582)] = 0.376$

$z = \frac{-1}{10}[3+(0.376)+6(0.376)] = -0.588$

3rd Iteration:-

$x = \frac{1}{10}[3+5(0.376)-2(-0.588)] = 0.606$

$y = \frac{1}{10}[3+4(0.606)+3(-0.588)] = 0.366$

$z = \frac{-1}{10}[3+0.606+6(0.366)] = -0.580$

4th Iteration:-

$x = \frac{1}{10}[3+5(0.366)-2(-0.580)] = 0.599$

$y = \frac{1}{10}[3+4(0.599)+3(-0.580)] = 0.366$

$z = \frac{-1}{10}[3+0.599+6(0.366)] = -0.580$

5th Iteration:-

$x = \frac{1}{10}[3+5(0.366)-2(-0.580)] = 0.599$

$y = \frac{1}{10}[3+4(0.599)+3(-0.580)] = 0.366$

$z = \frac{-1}{10}[3+0.599+6(0.366)] = -0.580$

2. Solve the following system of equation by Gauss seidel method correct to 3 decimal places. (19)

$$x+y+54z = 110$$
$$25x+6y-z = 85$$
$$6x+15y+2z = 72$$

Soln:

Given equations $x+y+54z = 110$
$25x+6y-z = 85$
$6x+15y+2z = 72$

Since, the given system is not diagonally dominent.

$$27x+6y-z = 85$$
$$6x+15y+2z = 72$$
$$x+y+54z = 110$$

Hence, it's diagonally dominent

$$|27| > |6|+|1|$$
$$|15| > |6|+|2|$$
$$|54| > |1|+|1|$$

$$x = \frac{1}{27}[85-6y+z]$$
$$y = \frac{1}{15}[72-6x-2z]$$
$$z = \frac{1}{54}[110-x-y]$$

1st Iteration:-

$$x = \frac{1}{27}[85-0+0] \quad (\because y=0, z=0)$$
$$x = 3.148$$
$$y = \frac{1}{15}[72-6(3.148)-2(0)] \quad (\because x=3.148, z=0)$$
$$y = 3.541$$

$$z = \frac{1}{54}[110 - 3.148 - 3.541]$$

$$z = 1.913$$

2nd Iteration:-

$$x = \frac{1}{27}[85 - 6(3.541) + 1.913]$$

$$x = 2.432$$

$$y = \frac{1}{15}[72 - 6(2.432) - 2(1.913)]$$

$$y = 3.5721$$

$$z = \frac{1}{54}[110 - 2.432 - 3.572]$$

$$z = 1.926$$

3rd Iteration:-

$$x = \frac{1}{27}[85 - 6(3.572) + 1.926]$$

$$x = 2.426$$

$$y = \frac{1}{15}[72 - 6(2.426) - 2(1.926)]$$

$$y = 3.573$$

$$z = \frac{1}{54}[110 - 2.426 - 3.573]$$

$$z = 1.926$$

4th Iteration:-

$$x = \frac{1}{27}[85 - 6(3.573) + 1.926]$$

$$x = 2.425$$

$$y = \frac{1}{15}[72 - 6(2.425) - 2(1.926)]$$

$$y = 3.573$$

$$z = \frac{1}{54}[110 - 2.425 - 3.573]$$

$$z = 1.926$$

//

Unit - V

Interpolation :- (Equal Intervals)

1. Gregory - Newton Forward Interpolation Formula :-

(or)

Newton's Forward Interpolation Formula :-

$$P_n(x) = P_n(x_0 + uh) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

Where $u = \frac{x - x_0}{h}$

2. Gregory - Newton Backward Interpolation Formula :-

(or)

Newton's Backward Interpolation Formula :-

$$P_n(x) = P_n(x_n + vh) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

Where $v = \frac{x - x_n}{h}$

1. Construct the table from the following data :-

x :	20	23	26	29
$f(x) = y$:	0.3420	0.3907	0.4384	0.4848

Soln:

Required Table

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
20	0.3420	0.0487	-0.0010	
23	0.3907	0.0477		-0.0008
26	0.4384	0.0464	-0.0013	
29	0.4848			

Note:- 1

Error in polynomial Interpolation. If $y = f(x)$ is the exact cathe and $y = P_n(x)$ is the interpolating polynomial call curve the error in polynomial Interpolation is

$$\text{Error} = f(x) - P_n(x) = \frac{(x-x_0)(x-x_1)\cdots\cdots(x-x_n)}{(n+1)!} f^{(n+1)}(c).$$

Where $x_0 < x < x_n$ and $x_0 < c < x_n$.

Note: 2

Error in Newton's forward Interpolation Formula.

$$\text{Error} = f(x) - P_n(x) = \frac{u(u-1)(u-2)\cdots(u-n)}{(n+1)!} h^{n+1} f^{(n+1)}(c).$$

Where $u = \frac{x-x_0}{h}$

Note 13

Error in Newton's backward Interpolation Formula.

$$\text{Error} = f(x) - P_n(x) = \frac{v(v+1)(v+2)\ldots(v+n)}{(n+1)!} h^{n+1} f^{(n+1)}(c)$$

When $v = \frac{x - x_n}{h}$

Problem:-

1. Find the value of y at $x = 21$ and $x = 28$ from the following data.

x	20	23	26	29
$f(x) = y$	0.3420	0.3907	0.4384	0.4848

Soln:-

Since $x = 21$ is nearer to the beginning of the table, we use Newton's forward interpolation Formula.

Forward table:-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	0.3420 y_0	Δy_0 0.0487	$\Delta^2 y_0$ -0.0010	$\Delta^3 y_0$ -0.0003
23	0.3907	0.0477	-0.0013	
26	0.4384	0.0464		
29	0.4848			

WKT:-

Newton's forward Interpolation Formula is given by,

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots \quad \text{(1)}$$

Since $u = \frac{x - x_0}{h} = \frac{21-20}{3} = \frac{1}{3}$. $(\because x = 21)$ (24)

From (1),

$$y(x) = 0.3420 + \frac{1/3}{1}(0.0487) + \frac{1/3(1/3-1)}{2}(-0.0010) + \frac{1/3(1/3-1)(1/3-2)}{6}(-0.0003)$$

$$= 0.3420 + 0.0162 + 0.0002 - 0.0000$$

$$f(21) = 0.3583$$

Also, Since $x = 28$ is nearer to end value of the table, we use Newton's backward Interpolation Formula is given by,

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \ldots \quad \rightarrow (2)$$

Backward Table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
20	0.3420			
		0.0487		
23	0.3907		-0.0010	
		0.0477		-0.0003
26	0.4384		-0.0013	$\nabla^3 y_n$
		0.0464	$\nabla^2 y_n$	
29	0.4848 y_n	∇y_n		

Since $v = \frac{x - x_0}{h} = \frac{28-29}{3} = \frac{-1}{3}$ $(\because x = 28)$

from (2),

$$y(28) = 0.4848 + \frac{(-1/3)}{1}(0.0464) + \frac{(-1/3)(-1/3+1)}{2}(-0.0013)$$

$$+ \frac{(-1/3)(-1/3+1)(-1/3+2)}{6}(-0.0003)$$

$$= 0.4848 + (-0.0155) + (0.0001) + 0.0000$$

$$y(28) = 0.4695$$

1. 'A' population of a town is as follows

year (x) :	1941	1951	1961	1971	1981	1991
Population in Lakhs (y) :	20	24	29	36	46	51

Estimate the population increase during the period 1946 - 1976.

Proof:

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

Since the population of the year 1946.

ie., $x = 1946$

WKT: Newton's forward Interpolation formula is given by

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0 + \cdots\cdots$$

Here $u = \frac{x - x_0}{h} = \frac{1946 - 1941}{10} = \frac{5}{10} = 1/2$

$$y(1946) = 20 + \frac{1/2}{1}(4) + \frac{1/2(1/2-1)}{2}(1) + \frac{1/2(1/2-1)(1/2-2)}{6}(1) + \frac{1/2(1/2-1)(1/2-2)(1/2-3)}{24}(0) + \frac{1/2(1/2-1)(1/2-2)(1/2-3)(1/2-4)}{120}(-9)$$

$$= 20 + 2 + (-0.1250) + 0.0625 + 0 - 0.0461$$

$$y(1946) = 21.89$$

Also,

Since the population of the year 1976

ie., $x = 1976$

WKT: Newton's backward Interpolation formula is given by

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \frac{v(v+1)(v+2)(v+3)(v+4)}{5!}\nabla^5 y_n + \cdots\cdots$$

Here $v = \frac{x - x_n}{h} = \frac{1976 - 1991}{10} = \frac{-15}{10} = -3/2$

$$y(1976) = 51 + \frac{(-3/2)}{1}(5) + \frac{-3/2\,(-3/2+1)}{2}(-5) +$$

$$\frac{-3/2\,(-3/2+1)(-3/2+2)}{6}(-8) + \frac{-3/2\,(-3/2+1)(-3/2+2)(-3/2+3)}{24}(-9)$$

$$\frac{-3/2\,(-3/2+1)(-3/2+2)(-3/2+3)(-3/2+4)}{120}(-9)$$

$$= 51 - 7.5 - 1.8750 - 0.5000 - 0.2109 - 0.1055$$

$$y(1976) = 40.8086$$

∴ Increase the population, during the period between 1946 and 1976 is

2. From the data given below, find the number of student whose weight between 60 and 70.

Weight x :	0-40	40-60	60-80	80-100	100-120
Number of student y :	250	120	100	70	50

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	250				
		120			
60	120		-20		
		100		-10	
80	100		-30		20
		70		10	
100	70		-20		
		50			
120	50				

Since the interpolation $x = 70$

WKT; Newton's Forward Interpolation Formula is given by,

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

Since $u = \frac{x - x_0}{h} = \frac{60-40}{20} = \frac{20}{20} = 1$

$$y(x) = 250 + \frac{1}{1!}120 + \frac{1(1-1)}{2!}(-20) + \frac{1(1-1)(1-2)}{6}(-10) + \frac{1(1-1)(1-2)(1-3)}{24}(0)$$

$$= 250 + 120 + 0 + 0 + 0$$

$$y(x) = 370$$

Also,

Since the interpolation $x = 70$

WKT: Newton's Backward Interpolation Formula is given by,

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n + \cdots$$

$$v = \frac{70-120}{20} = \frac{-50}{20} = -2.5 \qquad v = -2.5$$

$$y(x) = 590 - \frac{2.5}{1}(50) + \frac{(-2.5)(-2.5+1)}{2}(-20) + \frac{-2.5(-2.5+1)(-2.5+2)}{6}(10) + \frac{-2.5(-2.5+1)(-2.5+2)(-2.5+3)}{24}(0)$$

$$= 590 - 125 + \frac{(-2.5)(-1.5)}{2}(-20) + \frac{(-2.5)(-1.5)(-0.5)}{6}$$

$$= 590 - 125 - \frac{75}{2} - \frac{18.75}{6}$$

$$= 590 - 125 - 37.5 - 3.1250$$

$$y(x) = 424$$

$$= -370 + 424$$

$$= 54 \text{ //.}$$

Interpolation with Unequal Intervals :-

Lagrange's Interpolation Formula :-

Let $y = f(x)$ be a function such that the value of independent variable are not equaly space

We will use lagrange's interpolation formula which is given by,

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)} f(x_1) + \ldots\ldots + \frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots(x_n-x_{n-1})} f(x_n)$$

Problem :-

1. Using Lagrange's Interpolation Formula find $y(10)$ from the following data.

x	5	6	9	11
$f(x)$	12	13	14	16

(-2543)

Soln:

WKT: The Lagrange's Interpolation Formula is given by,

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1)$$

$$+ \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)$$

Given that

x	5	6	9	11
$f(x)$	12	13	14	16

Now,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

When $x = 10$,

$$f(10) = y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13)$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}(16)$$

$$y(10) = \frac{4 \cdot 1 \cdot (-1)}{(-1)(-4)(-6)}(12) + \frac{5 \cdot 1 \cdot (-1)}{1 \cdot (-3)(-5)}(13) + \frac{5 \cdot 4 \cdot (-1)}{4 \cdot 3 \cdot (-2)}(14)$$

$$+ \frac{5 \cdot 4 \cdot 1}{6 \cdot 5 \cdot 2}(16)$$

$$= 2 - \frac{13}{3} + \frac{35}{8} + \frac{16}{3} = \frac{6 - 13 + 35 + 16}{3} = \frac{44}{3}$$

Hence $y(10) = 14.666$

2. Using Lagrange's Interpolation Formula find $y(9.5)$ given the following data

x	7	8	9	10
y	3	1	1	9

Soln:

WKT: The Lagrange's Interpolation Formula is given by

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

Given that

x	7	8	9	10
y	3	1	1	9

When $x = 9.5$

$$f(9.5) = y(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)}(3) + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)}$$

$$(1) + \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)}(1) + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)}(9)$$

$$y(9.5) = \frac{(1.5)(0.5)(-0.5)}{(-1)(-2)(-3)}(3) + \frac{(2.5)(0.5)(-0.5)}{1(-1)(-2)}(1)$$

$$+ \frac{(2.5)(1.5)(-0.5)}{2 \cdot 1 \cdot (-1)}(1) + \frac{(2.5)(1.5)(0.5)}{3 \cdot 2 \cdot 1}(9)$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.8125$$

$\therefore$ Hence $y(9.5) = 3.6250$

Numerical Integration :-

WKT :- $\int_a^b f(x)\,dx$ represent the area between $y = f(x)$, x axis and the ordinate $x = a$ and $x = b$.

This integration is possible only if the $f(x)$ is explicitly given and if it is integrable.

Trapezoidal Rule :-

WKT :

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

This is know as Trapezoidal rule

Note :-

The error in the Trapezoidal rule is of the order h^2.

1. Evaluate $\int_{-3}^{3} x^4\,dx$ by using Trapezoidal rule with $h = 1$

verify your results by actual Integration

Soln :-

Let $f(x) = x^4$ Here $a = -3$ and $b = 3$.

WKT:

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right] \rightarrow (1)$$

∴ Required Data is,

x	:	-3	-2	-1	0	1	2	3
$f(x)$	:	81	16	1	0	1	16	81
		y_0	y_1	y_2	y_3	y_4	y_5	y_6

From (1),

$$\int_{-3}^{3} x^4\,dx = \frac{1}{2}\left[(81+81)+2(16+1+0+1+16)\right]$$

$$= 1/2\,[162+2(34)]$$

$$= 115 \rightarrow (2)$$

By usual Integration

WKT:

$$\int_{-3}^{3} x^4\,dx = \frac{1}{5}\left[(3)^5-(-3)^5\right] = \frac{1}{5}[243+243]$$

$$= 1/5\,[486] = 97.2 \rightarrow (3)$$

2. Evaluate $I = \int_0^6 \frac{1}{1+x}\,dx$ using Trapezoidal rule, also checkup by directe integration.

2.0214

actu 1.9459

WKT:

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

$f(x) = \frac{1}{1+x}$ Here $a=0$ and $b=6$

WKT:

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

$\hookrightarrow (1)$

Required data is,

x	:	0	1	2	3	4	5	6
$f(x)$	:	1 (y_0)	1/2 (y_1)	1/3 (y_2)	1/4 (y_3)	1/5	1/6 (y_{n-1})	1/7 (y_n)

from ①,

$$\int_0^6 \frac{1}{1+x}\,dx = \frac{1}{2}\left[(1+1/7) + 2(1/2+1/3+1/4+1/5+1/6)\right]$$

$$= \frac{1}{2}\left[(1.1429) + 2(0.5000+0.3333+0.2500 + 0.2000+0.1667)\right]$$

$$= \frac{1}{2}\left[(1.1429)+2(1.4500)\right]$$

$$= \frac{1}{2}\left[1.1429+2.9000\right] = \frac{1}{2}\left[4.0429\right]$$

$$= 2.0215$$

By actual Integration

WKT: $\int_0^6 \frac{1}{1+x}dx = \left[\frac{-2}{1+x^2}\right]_0^6$

$$= \left[\frac{-2}{1+6^2} + \frac{2}{1+0^2}\right]$$

$$= \left[\frac{-2}{37} + \frac{2}{1}\right]$$

$$= -0.0541+2$$

$$\int_0^6 \frac{1}{1+x}\,dx = 1.9459$$

Simpson's one-Third (1/3) Rule :-

WKT:-

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

Note:-

The error in simpson's one-third rule is of order h^4.

Problem:-

1. Evaluate $I = \int_0^6 \frac{1}{1+x}\,dx$ using simpson's one-third rule check out by direcle Integration.

Soln:-

Take $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

Let $f(x) = \frac{1}{1+x}$

x	:	0	1	2	3	4	5	6
$f(x)$	:	1	1/2	1/3	1/4	1/5	1/6	1/7

WKT:-

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

$$\int_0^6 \frac{1}{1+x}\,dx = 1/3\left[(1+1/7) + 2(1/3+1/5) + 4(1/2+1/4)\right]$$

$$= 1/3\left[1.1429 + 2(0.5333) + 4(0.7500)\right]$$

$= 1/3 [1.1429 + 1.0666 + 3.0000]$

$= 1/3 [5.2095]$

$= 1.7365$

By actual Integration

$$\int_0^6 \frac{1}{1+x}\,dx = \int_0^6 \frac{d(1+x)}{1+x} = [\log(1+x)]_0^6$$

$= \log 7 - \log 1$

$= \log_e 7 = 1.9459$

2. Dividing the range into 10 equal parts

Evaluate $\int_0^{\pi} \sin x\,dx$

$$h = \frac{b-a}{n} = \frac{\pi - 0}{10} = \pi/10$$

Let $f(x) = \frac{1}{1+x} \sin x$

x :	0	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$
$f(x)$:	0	0.3090	0.5878	0.8090	0.9511	1.0000	0.9511

	$7\pi/10$	$8\pi/10$	$9\pi/10$	π
	0.8090	0.5878	0.3090	0

Trapezoidal Rule:-

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\ldots\ldots+y_{n-1})\right]$$

from ①,

$$\int_0^{\pi} \sin x\,dx = \frac{\pi/20}{2}\left[(0+0)+2(0.3090+0.5878+0.8090+0.9511+1.0000+0.9511+0.8090+0.5878+0.3090)\right]$$

$$= \frac{\pi}{20}\left[2(6.3138)\right]$$

$$= \frac{\pi}{20}[12.6276] = \frac{3.14}{20}[12.6276]$$

$$= 1.9825$$

Simpson's one-third rule:-

$$\int_{x_0}^{x_n} f(x) = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+y_6+\ldots)+4(y_1+y_3+y_5+\ldots)\right]$$

$$= \frac{\pi/10}{3}\left[(0+0)+2(0.5878+0.9511+0.9511+0.5878)+4(0.3090+0.8090+1.0000+0.8090+0.3090)\right]$$

$$= \frac{\pi}{30}\left[2(3.0778)+4(3.2360)\right]$$

$$= \frac{\pi}{30}\left[6.1556+12.9440\right]$$

$$= \frac{3.14}{30}\left[19.0996\right]$$

Numerical solution of ordinary differential Equation:-

Euler's Method:-

To solve $\frac{dy}{dx} = f(x,y)$ with initial condition $y(x_0) = y_0$, then the Euler formula is given by

$$y_{n+1} = y_n + h f(x_n, y_n), \quad n = 0, 1, 2, \ldots$$

Put $n=0$,

$$y_1 = y_0 + h f(x_0, y_0)$$

Put $n=1$,

$$y_2 = y_1 + h f(x_1, y_1)$$

and so on,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

This formula is called Euler's Algorithm.

1. Using Euler's method solve numerically in the equation $y' = x+y$, $y(0)=1$ for $x = (0.0)(0.2)(1.0)$.

Soln:

Given $y' = x+y$ with $y(0)=1$ and $x = (0.0)(0.2)(1.0)$

Here $x_0 = 0.0$, $y_0 = 1$, $h = 0.2$

Let $f(x,y) = x+y \Rightarrow f(x_0, y_0) = x_0 + y_0 = 0+1 = 1$

WKT:-

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y_1 = y(0.2) = 1 + (0.2)(1) = 1 + 0.2 = 1.2$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$y_2 = y(0.4) = 1.2 + (0.2)(1.4) = 1.48$$

$y_3 = y_2 + h \cdot f(x_2, y_2)$

$= 1.48 + (0.2)(1.88) = 1.856$

$y_4 = y_3 + h \cdot f(x_3, y_3)$

$y_4 = y(0.8) = 1.856 + 0.2(2.456)$

$= 2.3472$

$y_5 = y_4 + h\, f(x_4, y_4)$

$y_5 = y(1.0) = 2.3472 + 0.2(3.1472)$

$= 2.9766$

$f(x_1, y_1) = x_1 + y_1$

$= 0.2 + 1.2 \quad (\because x_1 = 0.2$

$= 1.4$

$(x_2 = 0.4$

$f(x_2, y_2) = x_2 + y_2$

$= 0.4 + 1.48$

$= 1.88$

$(x_3 = 0.6$

$f(x_3, y_3) = x_3 + y_3$

$= 0.6 + 1.856$

$= 2.4560$

$(x_4 = 0.8$

$f(x_4, y_4) = x_4 + y_4$

$= 3.1472$

2. Given $y' = -y$ and $y(0) = 1$. determined the value of y at $x = (0.01)\,(0.01)\,(0.04)$

(x_1, h, x_n)

$y_1 = 0.99$

$y_2 = 0.9801$

$y_3 = 0.9703$

$y_4 = 0.9606$

Runge-Kutta Method:-

Forth order Runge-Kutta Method:-

1. To solve $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$.

WKT:- $y(x+h) = y(x) + \Delta y$

where $\Delta y = \frac{1}{6}[K_1 + 2(K_2 + K_3) + K_4]$

Here $K_1 = h\, f(x, y)$

$k_2 = h \cdot f\left(x + h/2, y + \frac{k_1}{2}\right)$

$k_3 = h\, f\left(x + h/2, y + \frac{k_2}{2}\right)$

$k_4 = hf(x+h, y+k_3)$

(or)

$$y(x+h) = y(x) + \frac{1}{6}[k_1 + 2(k_2+k_3) + k_4]$$

2. **Soln:**

Given $y' = -y$ with $y(0) = 1$ and $x = (0.01)(0.01)(0.04)$

Here $x_1 = 0.01$; $y_0 = 1$, $h = 0.01$

Let $f(x,y) = -y \Rightarrow f(x_0, y_0) = -y_0 = -1$

WKT:

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 1 + (0.01)(-1) = 1 - 0.01 = 0.9900$

$y_2 = y_1 + hf(x_1, y_1)$

$= 0.9900 + (0.01)(-0.9900)$

$f(y_1) = -y_1 = -0.9900$

$f(y_2) = -y_2 = -0.9801$

$f(y_3) = -y_3 = -0.9703$

$y_2 = 0.9900 - 0.0099 = 0.9801$

$y_3 = y_2 + h f(x_2, y_2)$

$= 0.9801 + (0.01)(-0.9801)$

$y_3 = 0.9801 - 0.0098 = 0.9703$

$y_4 = y_3 + h f(x_3, y_3)$

$= 0.9703 + (0.01)(-0.9703)$

$y_4 = 0.9703 - 0.0097 = 0.9606$

1. Apply the 4th order Runge-kutta Method to find $y(0.2)$ given that $y' = x+y$ with $y(0) = 1$. (41)

Soln:

Given $\frac{dy}{dx} = x+y$, $y(0) = 1$ Here $x_0 = 0, y_0 = 1$.

WKT: Fourth order Runge-kutta method Formula is given by, To find $y(0.2)$.

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2(k_2+k_3) + k_4] \longrightarrow (1)$$

$\left(\because y(x+h) = y(x) + \frac{1}{6}[k_1+2k_2+2k_3+k_4]\right.$

$x = x_0$,

$\therefore y_1 = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$\therefore h = 0.2$

We have,

$$k_1 = hf(x_0, y_0)$$

$$= (0.2)(1)$$

$$k_1 = 0.2$$

$$k_2 = hf\left(x_0 + h/2, y_0 + \frac{k_1}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$$

$$= 0.2 f(0.1, 1.1)$$

$$= 0.2[0.1 + 1.1] = 0.2(1.2)$$

$$k_2 = 0.2400$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.24}{2}\right)$$

$$= 0.2 f(0.1, 1.12)$$

$$= 0.2[0.1 + 1.12] = 0.2[1.22]$$

$$k_3 = 0.2440$$

$k_4 = h f(x_0+h, y_0+k_3)$

$= 0.2\, f(0+0.2, 1+0.2440)$

$= 0.2\,[0.2+1.2440]$

$= 0.2\,[1.4440]$

$k_4 = 0.2888$

From,

$y_1 = y(0.2) = 1 + \frac{1}{6}[0.2+2(0.24+0.2440)+0.2888]$

$= 1 + \frac{1}{6}[1.4568] = 1 + 0.2428$

$= 1.2428$

2. Using Runge-kutta Method of 4th order, find $y(0.8)$ correct to 4 decimal places if $y' = (y-x^2)$, $y(0.6) = 1.7379$

Soln

Given $\frac{dy}{dx} = y - x^2$, $x_0 = 0.6$, $y_0 = 1.7379$, $h = 0.2$

$k_1 = h f(x_0, y_0)$

$= 0.2\, f(0.6, 1.7379)$

$= 0.2\,[1.7379 - (0.6)^2]$

$= 0.2\,[1.7379 - 0.3600] = 0.2\,[1.3779]$

$k_1 = 0.2756$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2\, f\left[0.6 + \frac{0.2}{2}, 1.7379 + \frac{0.2756}{2}\right]$

$= 0.2\, f(0.7, 1.8757)$

$= 0.2\, [1.8757 - (0.7)^2]$

$= 0.2\, [1.8757 - 0.4900]$

$= 0.2\, [1.3857]$

$k_2 = 0.27771$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2\, f\left[0.6 + \frac{0.2}{2}, 1.7379 + \frac{0.27771}{2}\right]$

$= 0.2\, f(0.7, 1.8765)$

$= 0.2\, [1.8765 - (0.7)^2]$

$= 0.2\, [1.8765 - 0.4900] = 0.2\, [1.3865]$

$k_3 = 0.27773$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.2\, f(0.6 + 0.2, 1.7379 + 0.27773)$

$= 0.2\, f(0.8, 2.0152)$

$= 0.2\, [2.0152 - (0.8)^2]$

$= 0.2\, [2.0152 - 0.6400] = 0.2\,(1.3752)$

$k_4 = 0.27750$

from,

$y_1 = y(0.8) = 1.7379 + \frac{1}{6}\,[0.27756 + 2(0.27771 + 0.27773) + 0.27750]$

$= 1.7379 + \frac{1}{6}\,[1.6594]$

$= 1.7379 + 0.2766$

$= 2.0145$

1. compute $y(0.3)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using R.K Method of 4th order with 4 decimal places.

Soln:

Given $\frac{dy}{dx} + y + y^2 x$, $y(0) = 1$, $h = 0.1$, $y(0.3) = ?$

ie., $\frac{dy}{dx} = -(y + y^2 x)$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

WKT:-

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \rightarrow (1)$$

Now,

$$k_1 = hf(x_0, y_0) = 0.1[1 + 1^2(0)] = -(0.1)(1) = -0.1$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.1 f\left[0 + \frac{0.1}{2}, 1 - \frac{0.1}{2}\right].$$

$$= 0.1 f(0.05, 0.95)$$

$$= -0.1[0.95 + (0.95)^2 (0.05)]$$

$$= -0.1[0.9951]$$

$$k_2 = -0.0995$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{(-0.0995)}{2}\right)$$

$$= 0.1 f(0.05, 0.9503)$$

$$= -0.1[0.9503 + (0.9503)^2 (0.05)]$$

$= -0.1\,(0.9955)$

$k_3 = -0.0995$

$k_4 = hf(x_0+h, y_0+k_3)$

$= 0.1 f(0+0.1, 1+(-0.0995))$

$= 0.1 f(0.1, 0.9005)$

$= -0.1\,[0.9005+(0.9005)^2(0.1)]$

$k_4 = -0.0982$

from ①,

$y_1 = y(0.1) = 1+\frac{1}{6}[-0.1+2(-0.0995)+2(-0.0995)-0.0982]$

$= 1+\frac{1}{6}[-0.1-0.1990-0.1990-0.0982]$

$= 1-\frac{1}{6}(0.5962)$

$\boxed{y_1 = 0.9006}$

Also, $y_2 = y(0.2) = y_1+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$, → ②

$k_1 = hf(x_1, y_1)$

$x_1 = x_0+h$
$= 0+0.1$
$x_1 = 0.1$, $y_1 = 0.9006$

$= 0.1 f(0.1, 0.9006)$

$= -0.1\,[0.9006+(0.9006)^2(0.1)]$

$k_1 = -0.0982$

$k_2 = hf\left[x_1+\frac{h}{2}, y_1+\frac{k_1}{2}\right]$

$= 0.1 f\left[0.1+\frac{0.1}{2}, 0.9006+\frac{(-0.0982)}{2}\right]$

$= -0.1 f(0.15, 0.8515)$

$= -0.1\,[0.8515+(0.8515)^2(0.15)]$

$k_2 = -0.0960$

$k_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}\right)$

$= 0.1\, f\left[0.1+\frac{0.1}{2},\ 0.9006+\frac{(-0.0960)}{2}\right]$

$= 0.1\, f(0.15, 0.8526)$

$= -0.1\,[0.8526+(0.8526)^2(0.15)]$

$k_3 = -0.0962$

$k_4 = hf(x_1+h, y_1+k_3)$

$= 0.1\, f(0.1+0.1,\ 0.9006+(-0.0962))$

$= 0.1\, f(0.2, 0.8044)$

$= -0.1\,[0.8044+(0.8044)^2(0.2)]$

$k_4 = -0.0934$

from ②,

$y_2 = y(0.2) = 0.9006+\frac{1}{6}[-0.0982+2(-0.0960)+2(-0.0962)-0.0934]$

$= 0.9006+\frac{1}{6}[-0.0982-0.1920-0.1924-0.0934]$

$= 0.9006-\frac{1}{6}[0.5760]$

$\boxed{y_2 = 0.8046}$

Also, $y_3 = y_2+\frac{1}{6}[k_1+2(k_2+k_3)+k_4]$ → ③

$x_2 = x_1+h$
$= 0.1+0.1$
$x_2 = 0.2$
$y_2 = 0.8046$

$k_1 = hf(x_2, y_2)$

$= 0.1\, f(0.2, 0.8046)$

$= -0.1\,[0.8046+(0.8046)^2(0.2)]$

$k_1 = -0.0934$

$k_2 = hf\left(x_2+\frac{h}{2}, y_2+\frac{k_1}{2}\right)$

$= 0.1 f\left[0.2+\frac{0.1}{2}, 0.8046+\frac{(-0.0934)}{2}\right]$

$= 0.1 f(0.25, 0.7579)$

$= -0.1\left[0.7579+(0.7579)^2(0.25)\right]$

$k_2 = -0.0902$

$k_3 = hf\left(x_2+\frac{h}{2}, y_2+\frac{k_2}{2}\right)$

$= 0.1 f\left[0.2+\frac{0.1}{2}, 0.8046+\frac{(-0.0902)}{2}\right]$

$= 0.1 f(0.25, 0.7595)$

$= -0.1\left[0.7595+(0.7595)^2(0.25)\right]$

$k_3 = -0.0904$

$k_4 = hf(x_2+h, y_2+k_3)$

$= 0.1 f[0.2+0.1, 0.8046+(-0.0904)]$

$= 0.1 f(0.3, 0.7142)$

$= -0.1\left[0.7142+(0.7142)^2(0.3)\right]$

$k_4 = -0.0867$

from ③,

$y_3 = y(0.3) = 0.8046+\frac{1}{6}[-0.0934+2(-0.0902)+2(-0.0904)-0.0867]$

$= 0.8046-\frac{1}{6}[0.0934+0.1804+0.1808+0.0867]$

$= 0.8046-\frac{1}{6}(0.5413)$

$= 0.8046-0.0902$

$\boxed{y_3 = 0.7144}$

2. Using R.K. Method of 4th order solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ given $y(0)=1$ at $x=0.2, 0.4$.

(48)

Soln:

Given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0)=1$, $x_0=0$, $y_0=1$, $h=0.2$

$$y_1 = y_0 + \frac{1}{6}[K_1+2(k_2+k_3)+k_4] \rightarrow (1)$$

$K_1 = hf(x_0, y_0)$

$= 0.2 f(0,1)$

$= 0.2 \left[\frac{1^2-0^2}{1^2+0^2}\right]$

$K_1 = 0.2$

$K_2 = 0.2 hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$

$= 0.2 f\left(0+\frac{0.2}{2}, 1+\frac{0.2}{2}\right)$

$= 0.2 f(0.1, 1.1)$

$= 0.2 \quad \frac{(1.1)^2-(0.1)^2}{(1.1)^2+(0.1)^2}$

$= 0.2 \quad \frac{1.21-0.01}{1.21+0.01}$

$K_2 = 0.1967$

$K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$

$= 0.2 f\left(0+\frac{0.2}{2}, 1+\frac{0.1967}{2}\right)$

$= 0.2 f(0.1, 1.0984)$

$= 0.2 \quad \frac{(1.0984)^2-(0.1)^2}{(1.0984)^2+(0.1)^2}$

$$= 0.2 \quad \frac{1.2065 - 0.01}{1.2065 + 0.01}$$

$k_3 = 0.1967$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 f(0 + 0.2, 1 + 0.1967)$

$= 0.2 f(0.2, 1.1967)$

$= 0.2 \quad \frac{(1.1967)^2 - (0.2)^2}{(1.1967)^2 + (0.2)^2}$

$k_4 = 0.1891$

from ①,

$y_1 = y(0.2) = 1 + \frac{1}{6}[0.2 + 2(0.1967) + 2(0.7967) + 0.1891]$

$= 1 + \frac{1}{6}[1.1759]$.

$\boxed{y_1 = 1.1960}$

$x_1 = x_0 + h$
$= 0 + 0.2$
$x_1 = 0.2$
$y_1 = 1.1960$

Also, $y_2 = y_1 + \frac{1}{6}[k_1 + 2(k_2 + k_3) + k_4] \rightarrow ②$

$k_1 = hf(x_1, y_1)$

$= 0.2 f(0.2, 1.1960)$

$= 0.2 \quad \frac{(1.1960)^2 - (0.2)^2}{(1.1960)^2 + (0.2)^2} = 0.2 \quad \frac{1.3904}{1.4704}$

$k_1 = 0.1891$

$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$

$= 0.2 f\left(0.2 + \frac{0.2}{2}, 1.1960 + \frac{0.2}{2}\right)$

$= 0.2 f(0.3, 1.2960)$

$= 0.2 \quad \frac{(1.2960)^2 - (0.3)^2}{(1.2960)^2 + (0.3)^2}$

$k_2 = 0.1797$

$$k_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}\right)$$

$$= 0.2f\left[0.2+\frac{0.2}{2}, 1.1960+\frac{0.1797}{2}\right]$$

$$= 0.2f(0.3, 1.2859)$$

$$= 0.2\ \frac{(1.2859)^2-(0.3)^2}{(1.2859)^2+(0.3)^2}$$

$$k_3 = 0.1794$$

$$k_4 = hf(x_1+h, y_1+k_3)$$

$$= 0.2f(0.2+0.2;\ 1.1960+0.1794)$$

$$= 0.2f(0.4, 1.3754)$$

$$= 0.2\ \frac{(1.3754)^2-(0.4)^2}{(1.3754)^2+(0.4)^2}$$

$$= 0.2\ \frac{1.7317}{2.0517}$$

$$k_4 = 0.1688$$

$$y_2 = y(0.4) = 1.1960+\frac{1}{6}[0.1891+2(0.1797)+2(0.1794)+0.1688]$$

$$= 1.1960+\frac{1}{6}[1.0761]$$

$$\boxed{y_2 = 1.3754}$$

Unit - IV

Curve Fitting:

The curve fitting we have two method,

1. The graphical method.
2. The method of group average.

The evaluation of constant vering from one grouping to another grouping of data. So, we adopt another method called the method of least squares.

Which gives a unique set of values to the constant in the equation of the fitting curve.

Fitting a straight line by the method of least square:

If $y = ax + b$ be the straight line then the normal equations are

$$a\Sigma x + nb = \Sigma y$$
$$a\Sigma x^2 + b\Sigma x = \Sigma xy$$

1. By method of least square find the best fitting straight line to the data given below

x	5	10	15	20	25
y	15	19	23	26	30

Soln:

WKT:- Equation of the straight line is $y = ax + b$.

Then the normal equations are $a\Sigma x + nb = \Sigma y$

$$a\Sigma x^2 + b\Sigma x = \Sigma xy$$

Here $n = 5$, The normal equation becomes

$$a\Sigma x + 5b = \Sigma y \quad \rightarrow (1)$$
$$a\Sigma x^2 + b\Sigma x = \Sigma xy \quad \rightarrow (2)$$

To calculate $\Sigma x, \Sigma x^2, \Sigma xy$

Required Table

x	y	x^2	xy
5	15	25	75
10	19	100	190
15	23	225	345
20	26	400	520
25	30	625	750
$\Sigma x = 75$	$\Sigma y = 113$	$\Sigma x^2 = 1375$	$\Sigma xy = 1880$

from (1), $75a + 5b = 113 \rightarrow (3)$

from (2), $1375a + 75b = 1880 \rightarrow (4)$

(3) $\times 15 \Rightarrow$ $1125a + 75b = 1695$

$1375a + 75b = 1880$

$-250a = -185$

$$a = 0.74$$

$a = 0.74$ is substitute in equation (3)

$75(0.74) + 5b = 113$

$55.5 + 5b = 113$

$5b = 57.5$

$$b = 11.5$$

Hence, the best fitting straight is $y = 0.74x + 11.5$

2. Fit a straight line to the data given below also the estimate the value of y at $x = 2.5$

(53)

x :	0	1	2	3	4
y :	1	1.8	3.3	4.5	6.5

Soln:

WKT:- Equation of the straight line is $Y = ax + b$.

Then the normal equations are $a\Sigma x + nb = \Sigma y$

$a\Sigma x^2 + b\Sigma x = \Sigma xy$

Here $n = 5$, The normal equation becomes

$a\Sigma x + 5b = \Sigma y \rightarrow ①$

$a\Sigma x^2 + b\Sigma x = \Sigma xy \rightarrow ②$

Required Table

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.5	16	25.2
$\Sigma x = 10$	$\Sigma y = 16.9$	$\Sigma x^2 = 30$	$\Sigma xy = 47.1$

from ①, $10a + 5b = 16.9 \rightarrow ③$

from ②. $30a + 10b = 47.1 \rightarrow ④$

③ × 2 ⇒ $20a + 10b = 33.8$

$30a + 10b = 47.1$

$-10a = -13.3$

$\boxed{a = 1.33}$

$a = 1.33$ is substitute in equation ③

$10a + 5b = 16.9$

$10(1.33) + 5b = 16.9$

$13.3 + 5b = 16.9$

$5b = 3.60$

$\boxed{b = 0.72}$

$y = 1.33(2.5) + 0.72$

$= 3.3250 + 0.72$

$\therefore \boxed{y = 4.0450}$

Fitting a Parabola or Fitting a second degree curve by the Method of Least Square:-

Result:

The normal equations of Parabola or second degree are $a\Sigma x^2 + b\Sigma x + nc = \Sigma y$.

$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy$

$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y$

4. Fitting a Parabola by the method of least square to also estimate the following data also estimate when $x=6$.

x	1	2	3	4	5
y	5	12	26	60.97	

(55)

Proof:

Let $y = ax^2 + bx + c$ be the best fit.

WKT: The normal equations are, $n=5$

$$a\Sigma x^2 + b\Sigma x + 5c = \Sigma y \rightarrow (1)$$

$$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy \rightarrow (2)$$

$$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y \rightarrow (3)$$

x	y	xy	x^2	x^3	x^4	x^2y
1	5	5	1	1	1	5
2	12	24	4	8	16	48
3	26	78	9	27	81	234
4	60	240	16	64	256	960
5	97	485	25	125	625	2425
$\Sigma x = 15$	$\Sigma y = 200$	$\Sigma xy = 832$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma x^2y = 3672$

from (1), $55a + 15b + 5c = 200 \rightarrow (4)$

from (2), $225a + 55b + 15c = 832 \rightarrow (5)$

from (3), $979a + 225b + 55c = 3672 \rightarrow (6)$

(4) $\times 3 \Rightarrow$ $165a + 45b + 15c = 600$

(5) $\Rightarrow$ $225a + 55b + 15c = 832$

$$-60a - 10b = -232$$

$$60a + 10b = 232 \rightarrow (7)$$

④ × 11 ⇒ $605a + 165b + 55c = 2200$

⑤ ⇒ $979a + 225b + 55c = 3672$

$-374a - 60b = -1472$

$374a + 60b = 1472 \rightarrow ⑧$

⑦ × 6 ⇒ $366a + 60b = 1392$

⑧ ⇒ $374a + 60b = 1472$

$-14a = -80$

$a = 80/14 = 5.7143$

a-ன் மதிப்பை ⑦-ல் பிரதியிட....

$60(5.7143) + 10b = 232$

$342.8580 + 10b = 232$

$10b = 232 - 342.8580$

$b = -11.0858$

a, b-ன் மதிப்பை ④-ல் பிரதியிட....

$55(5.7143) + 15(-11.0858) + 5c = 200$

$314.3140 - 160.2870 + 5c = 200$

$148.0270 + 5c = 200$

$c = \frac{51.9730}{5}$

$c = 10.3946$

$y = 5.7143x^2 - 11.0858x + 10.3946$, $x = 6$ எனில்

$= 5.7143(36) - 11.0858(6) + 10.3946$

$= 205.7148 - 66.5148 + 10.3946$

$= 216.1094 - 66.5148$

$y = 149.5946$

(2). Fit a second degree Parabola to the following data taking y is depending variable.

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

(57)

soln:

Let $y = a^2 + bx + c$ be the best fit.

WKT:- The normal equations are, $n = 9$

$$a\Sigma x^2 + b\Sigma x + 9c = \Sigma y \rightarrow (1)$$

$$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy \rightarrow (2)$$

$$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y \rightarrow (3)$$

x	y	xy	x^2	x^2y	x^3	x^4
1	2	2	1	2	1	1
2	6	12	4	24	8	16
3	7	21	9	63	27	81
4	8	32	16	128	64	256
5	10	50	25	250	125	625
6	11	66	36	396	216	1296
7	11	77	49	539	343	2401
8	10	80	64	640	512	4096
9	9	81	81	729	729	6561
$\Sigma x = 45$	$\Sigma y = 74$	$\Sigma xy = 421$	$\Sigma x^2 = 285$	$\Sigma x^2 y = 2771$	$\Sigma x^3 = 2025$	$\Sigma x^4 = 15333$

$$285a + 45b + 9c = 74 \rightarrow (4)$$

$$2025a + 285b + 45c = 421 \rightarrow (5)$$

$$15333a + 2025b + 285c = 2771 \rightarrow (6)$$

~~④×⑤~~

④ × 5 ⇒ $1425a + 225b + 45c = 370$

⑤ ⇒ $2025a + 285b + 45c = 421$

$-600a - 60b = -51$

$600a + 60b = 51 \rightarrow ⑦$

④ × 285 ⇒ $81225a + 12825b + 2565c = 21090$

⑥ × 9 ⇒ $137997a + 18225b + 2565c = 24939$

$-56772a - 5400b = -3849$

$56772a + 5400b = 3849 \rightarrow ⑧$

⑧ ⇒ $56772a + 5400b = 3849$

⑦ × 90 ⇒ $54000a + 5400b = 4590$

$2772a = -741$

$a = -0.2673$

Put a value in ⑦

$600(-0.2673) + 60b = 51$

$-160.38 + 60b = 51$

$b = \frac{51 + 160.38}{60}$

$b = 3.523$

Put a and b value in (4)

$$285(-0.2673) + 45(3.5232) + 9c = 74$$

$$82.3545 + 9c = 74$$

$$c = -0.9282$$

$$-0.2673x^2 + 3.523x - 0.9282 = 0$$

(59)

Unit - I

Matlab Environment :-

Definition :-

Matlab is one of a number of commercialy available sophisticated mathematical compatation tool such as MAPLE, Mathematica & Mathcad.

Todays, MATLAB has capabilities for lean the original MATLAB and is an intractive system an programming language general scientific & technical computation. Its basic limit matrix, because MATLAB comments similar to the way that we express engineering steps in mathematics, writing computer solution in MATLAB can be much quicker than writing solution in a high level language.

It is usually easier to write a "number crunching" program in Matlab it is usually execute faster in C++, FORTRAN.

In one exception to this rule is with because MATLAB is

optimise for matrices if a problem can be formulated with a matrix solution, MATLAB execute substantially faster than a similar program in a high level language.

Student Edition of MATLAB:-

We explain the differents between the professional and student version of MATLAB and introduce you to the MATLAB environment.

A member of examples are presented

The MATH works both provides and inexpensive student version for the microsoft windows, MC Intosh and linux operating system. The

The student version of MATLAB include the following component are

(i). The basic MATLAB engine and development environment.

(ii). The MATLAB Notebook Simulink

(iii). The symbolicmath toolbox function.

Note:-

Only differents the only differents you should notice is the command prompt ' in the professional version the prompt is >> ,

in the student version it is EDU >> .

MATLAB Windows:-

TO begin MATLAB icon or the start menu. If you are using a Linux UNIX operating system, type matlab at the shell prompt.

You should see the MATLAB prompt (>> or EDU >>). Which tells you that MATLAB is waiting for you to enter the command.

TO exit MATLAB type quit or exit at the MATLAB prompt.

Note:-

Choose exit MATLAB from the file menu or select the close icon (x) from the upper right hand corner of the screen.

MATLAB uses several display windows include a large a command window on the write, left for the current directly

work space and command history windows. In addition, document windows, graphics windows and editing windows will automatically open when needed.

Command Window :-

If you want to save the command sequence, you will need to use the editing windows to create and M-file are MATLAB files that contain programming code. An

An M-file is an

Similar to a 'C' are or FORTRAN source code file.

Calculation in a command windows very Similar to a Scientific calculator. most of the Syntax is even the same.

To compute the value of the five Square (5^2).

Type command 5^2.

Output will display ans=25.

To find the value of cos(π).

Type cos(pi)

output will be ans = -1

Command History:

The command history window, recorded the command when you exeist ——— (or) When you exit MATLAB When you issue the clc command, the command window is clear.

Ex: The command window type clc. This should clear the command window but leave the data in the command history window intact.

Command window by double clicking or by clicking and dragging the line code into the command window.

Double clicking cos(Pi)

ans = -1

Click and drag 5^2 from the command history window into the command window. The command would not execute until you hit enter and then you will get. ans = 25.

Work Space Window:-

The work space window keeps drag of the variable you have defined as

you execute command in the command window.

The work space window should show one variable, ans, can tell as, it has a value of 25 and is a double array.

Name	Value	Class
ans	25	double array

The new MATLAB 7, display is.

Name	Value	Size	Bytes	Class
ans	25	1x1	8	double array

grid symbol indicates the variable ans is an array.

The size 1x1 tell as, it is a single value. It is called a scalar.

The array uses 8 bytes of memory an the class designation tell as is ans is a double precision, floating point array.

Note :-

MATLAB consider every member you enter to be a floating point member, whether you put a decimal in the number or not.

You can defined additional variable in the command window and they will be listed in the work space window.

For example $>> A = 5$

Which is written $A = 5$

The variable A has been added the work shop window, variable begenin. which list variable alphabatical order, variables begining with capital letter are listed first.

NAME	value	Size	Bytes	class
A	5	1X1	8	double Array
ans	25	1X1	8	double Array

Endering Matrices into MATLAB.

You can enter a simbol one dimentional matrix by typing $B = [1, 2, 3, 4]$ which written as $B = [1, 2, 3, 4]$

are optional.

The variable B has been added to the workspace window then

NAME	value	Size	Bytes	class
A	5	1x1	8	double Array
B	[1 2 3 4]	1x4	32	double Array
ans	25	1x1	8	double Array

You defined two dimentional matrices in a similar fashion, semicolon are used to separate rows.

C = [1 2 3 4 ; 10 20 30 40 ; 5 10 15 20]

which returns,

C =

1	2	3	4
10	20	30	40
5	10	15	20

C appear in the workspace window

NAME	value	Size	Bytes	Class
A	5	1x1	8	double Array
B	[1 2 3 4]	1x4	32	double Array
C	<3x4 double>	3x4	96	double Array

Current directly window :-

When MATLAB either owes files or sales information on to your computer, it uses the current directly the default the current directry where is depending on your version of the software and how it was installed.

The current directry can be changed by selecting another directry from the drop-down list located next to the directory directry listing.

Document window :-

Double clicking an any variable listed in the workspace window automatically launches a document window containing the array editor. Value store in the variable are displayed in a spreadsheet format. You can change the values in the array editor or you can add new values. Enter the

following command in the command window

```
C = [1 2 3 4 ; 10 20 30 40; 5 10 15 20 ];
```

placing a semicolon at the end of the command suppresses the output shows that it is not repeated back in the command window.

However see c should be listed in the workspace window double click it. A document window will open the above command window.

Graphics window :-

The Graphics windows launches automatically when you request the graph. To create a simple graph, create and array for x values.

```
x = [1 2 3 4 5];
```

Now create a list of y values

```
y = [10 20 30 40 50];
```

To create a graph use the plot command

plot (x, Y)

The graphics window opens automatically

MATLAB makes to easy to modify graphs by adding titles, x and Y labels, multiple lines and more.

Edit window:-

The Editing window is open by choosing File from the menu bar then New, and finally m-file. This window allows you to type and save a series of command without executing them.

To may also open the edit window by typing edit at the command prompt.

Start Button:-

The Start Button is located in the lower left hand corner of the MATLAB window, and various to MATLAB windows, help functions and internet Products, tool Boxes provide additional MATLAB functionality for specific content area.

Scalar operation:-

The arithmetic operation between Scalars is given by.

operation	Algebric Form	MATLAB Form
Addition	a+b	a+b
Subtraction	a-b	a-b
Multiplication	a×b	a*b
Division	a/b	a/b
Exponentiation	a^b	a^b

The command a=1+2. ↵

a is assigned a value 1+2 which is addition of two scalar quantity.

b has a value of 5.

b=5

Then x = a+b

will return,

x = 8

Note:-

A single equal sign (=) is known as in MATLAB as assignment operator.

Example:

x is assign as value of 8.

If you enter the variable name.

x

Into MATLAB x = 8

x is assign the value of 8, If we interpret assignment statement x = x+1.

The value stored in x was orginally 8. will return x = 9.

Precedence of Arithmetic operation :-

The precedence of Arithmetic operation performed in MATLAB

Precedence	operation
1.	Parentheses innermost first
2	Exponentiation Left to Right
3	Multiplication and division, Left to Right
4	Addition and Subtraction, Left to Right

Problem:

1. The basic equation discribing gas property is a ideal gas law $PV = nRT$ Where

P - Pressure, KPa

V - Volume, m^3

n - number of Kmoles of gas in the Sample

R - ideal gas constant 8.314 kPa m^3 / kmole k

T - Temperature, k

In addition the number of kmole of gas is equal to the mass of a gas divided by the mole mass. i.e. $n = m/mw$; where m - mass, kg

mw - molar mass, kg / kmole.

Soln:

Discribe The input and output

Input:-

$V = 1000\ m^3$

$T = 300\ K$

$P = 100\ kPa$

$mw = 29\ kg / kmole$

$R = 8.314\ kPa\ m^3 / kmole\ k$

output:-

____ mass: kg.

Develope a MATLAB Solution

```
P=100
P=100
T=300
T=300
V=1000
V=1000
```

MW = 29

mW = 29

R = 8.314

R = 8.314

n = (P*V) / (R * T)

n = 40.0930

m = n * mW

= 1.1627 e+003

$PV = nRT$

$\Rightarrow n = \frac{PV}{RT}$

$n = m/mW$

$\Rightarrow m = n * mW$

Array operations

The simplest way to defined a matrix is to use a list of numbers called an explicit list.

The command `X = [1 2 3 4]`

will returns,

```
X = 1 2 3 4
```

A new row is indicated by a semicolon so that the column vector is specified as `Y = [1 ; 2; 3 ; 4]`

will returns,

```
Y = 1
    2
    3
    4
```

A matrix that contains both rows and columns would be created in the statement

A = [1 2 3 4 ; 2 3 4 5 ; 3 4 5 6]

will returns,

$$A = \begin{matrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{matrix}$$

A number that is used frequently in chemistry is Avogadro's constant whose value to four significant digit

602, 200, 000, 000, 000, 000, 000, 000.

The diamedre of an iron atom is approchmently 140 Picometer which is 0.000000000140 meter.

Scientific notation express a value as a number between 1 and 10 multiply by a power of 10.

In scientific notation, Avogadro's number becomes 6.023×10^{23} and the diameter of the iron atom becomes 1.4×10^{-10}

In MATLAB.

in MATLAB, values in scientific notation are designed with an e between the Mantissa and the exponent.

For Example:-

Avogadro's Constant = 6.023 e23

Iron diameter = 140 e-12 or 1.4e-10

Display Formats-

When elements of a matrix of display in MATLAB.

Thus,

A = 5;

returns

A = 5

but

A = 5.1;

returns

A =

5.1000

and

A = 51.1

returns

A =

51.1000

MATLAB allows to specify to other

format. we use the command 'format long'.

A = 51.1

returns

A = 51.1000000000000

format short

A = 51.1

returns

A = 51.1000

MATLAB Command	Display	Example
format short	4 decimal digits	15.2345
format long	14 decimal digits	15.234533333333333
format short e	14 decimal digits	1.5234e+01
format long e	15 decimal digits	1.523453333333332e+01
format bank	2 decimal digits	15.23
format +	+, -, bank	+

Saving variables:-

TO Preserve the variables

to create the command window between sessions you must save the content of the workspace window to a file.

The default format is a binary file called a MAT-file.

To save the workspace to a file at the prompt type save <file_name>.

Save is a MATLAB command, file name is a user defined a file name.

The file you save will be stored in the current directory.

Type,

clear, clc

This will clear the workspace an the command window. verify, the workspace is empty by checking the workspace window.

Now defined several variables such as,

```
A = 5;
B = [1,2,3];
C = [1,2;3,4];
```

will returns,

A =

5

B =

1 2 3

C =

1 2
3 4

MATLAB Summary of the special characters and functions :

Special characters :

[ ] - forms matrices

( ) - used in statements to group operations used with a matrix name to identify specific elements.

, - seperates subscripts or matrix elements

; - separates rows in a matrix definition. Suppresses output when used in commands

: - used to generate matrices indicates all rows or all columns.

= - assignment operator - assigns a value to a memory location - not the same as an equality.

% - indicates a comment is an M-file

+ - Scalar and array addition

- - Scalar and array subtraction

* - Scalar multiplication

.* - Array multiplication

/ - Scalar division

./ - Array division

^ - Scalar exponentiation

.^ - Array exponentiation

Commands and functions:-

ans - default variable name for results of MATLAB calculations

clc - clear command screen

Clear - clears workspace

exit - terminates MATLAB

format + - Sets format to plus and minus signs only

format compact - Set format to compact form

format long - Sets format to 14 decimal places.

format long e - Sets format to 14 exponential places.

format loose - Sets format back to default, non compact form

format short - Sets format back to default, four decimal places.

format short e - Sets format to four exponential places

help - invokes help utility

linespace - linearly spaced vector function

load - loads matrices form a file

pi - numeric approximation of the value of π

quit - terminates MATLAB

save - saves variables in a file

who - lists variable in memory

whos - lists variable and their sizes.

Predefined MATLAB functions:

Elementary MATH functions:

The Elementary MATH functions include functions to perform a number of common

computation such as,

MATLAB Prefixed function	Actual value	Example
abs(x)	computes the absolute values of x.	abs(-3) ans = 3
sqrt(x)	computes the square root of x.	sqrt(85) ans = 9.2195
round(x)	Rounds x to the nearest integer	round(8.6) ans = 9
fix(x)	Rounds (or truncates) x to the nearest integer toward zero.	fix(8.6) ans = 8
floor(x)	Rounds x to the nearest integer toward - infinity	floor(-8.6) ans = -9
ceil(x)	Rounds x to the nearest integer toward + infinity	ceil(-8.6) ans = -8
sign(x)	Returns a value of -1 if x is less than zero, a value of 0 if x equals zero, and a value of +1 if x is greater than zero	sign(-8) ans = -1
rem(x,y)	computes the remainder of x/y	rem(25,4) ans = 1
exp(x)	computes the value of e^x, where e is the base for natural logarithms, or approximately 2.718282	exp(10) ans = 2.2026e+004
log(x)	computes the lnx(x), the natural logarithms of x to the base e.	log(10) ans = 2.3026
log10(x)	computes the $\log_{10}(x)$; The common logarithms of x to the base 10.	log10(10) ans = 1

Triconomentric Function:

The tricnomentric function assume that angle for represented an radians.

TO convert ** radians to degrees or degrees to radian use the following convert fact that $180^\circ = \pi$ radians

angles_degrees = angle_radians * (180/Pi):

angles_radians = angle_degrees * (Pi/180):

A complete list of trionomentric function available in MATLAB.

Sin(x) - computes the sine of x, where x is in radians

```
Sin(0)
ans=0
```

cos(x) - computes the cosine of x, where x is in radians

```
cos(Pi)
ans=-1
```

tan(x) - computes the tangent of x, where x is in radians

```
tan(Pi)
ans=-1.2246e-0
```

aSin(x) - computes the arc sine or inverse sine of, x where x must be between -1 and 1. The function returns an angle in radians between $\pi/2$ and $-\pi/2$.

```
aSin(-1)
ans=-1.570
```

Sinh(x) - computes the hyperbolic Sine of x, where x is in radians.

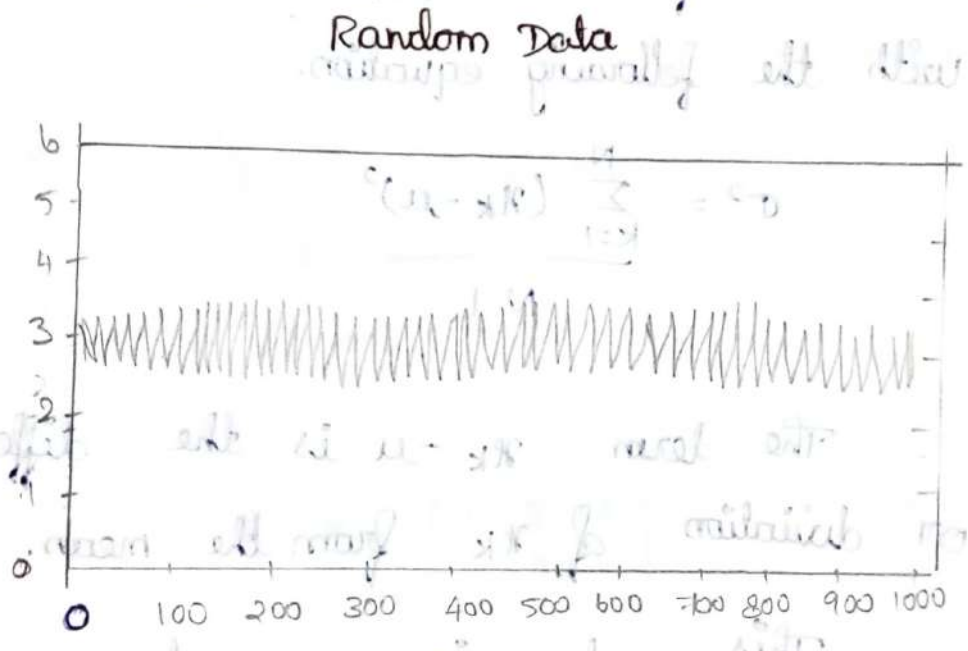
Sinh(Pi)

ans=11.5487

Variance and Standard Deviation :-

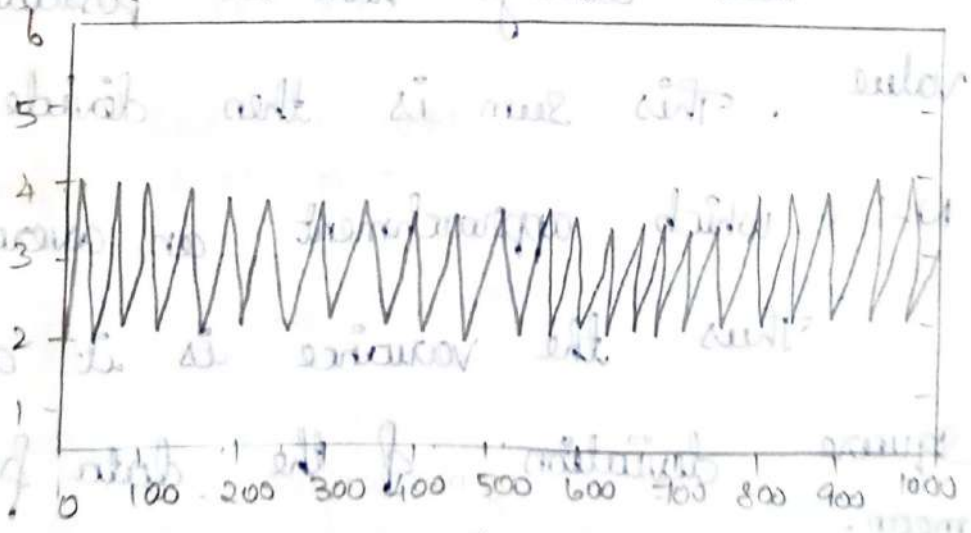
Two of the most important statistical measurements of a set of data are the variance and standard deviation.

Consider the values of vector



Mean=3.0

figure : 1



Mean=3.0

figure : 2

The data values in data-2 vary more or deviate more. Thus messures of variation and division from the values an data-2 will be greater than messure of variance and deviation for the data-1.

Mathematically the variance σ^2 for the set of data values can be computed with the following equation.

$$\sigma^2 = \frac{\sum_{k=1}^{N} (x_k - \mu)^2}{N-1}$$

The term $x_k - \mu$ is the difference or division of x_k from the mean.

This value is squared so that we will always have a possitive value. This sum is then divide by N-1, which approachment an average.

Thus the variance is it average squeare deviation of the data from the mean.

The standard diviation is defined as the square root of variance.

i.e.) $\sigma = \sqrt{\sigma^2}$

where σ is the Greek symbol sigma.

The syntax for calculating standard deviation in (MATLAB doesn't have a building function for variance) std(x)

Ex:1 if x = [1 5 3]

std(x)

Ans = 2

Ex:2

if $x = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

Input x = [1 5 3; 2 4 6]

std(x)

Ans = 0.7071
0.7071
2.1213

Random Numbers:-

Uniform Random Numbers:-

Random numbers that all equaly likely to be any value between an upper and lower limit our call uniform Random Numbers.

The rand function in MATLAB generate random numbers uniformly distributed over a interval 0 and 1.

rand(n)	As n×n matrix each values in the matrix is a random numbers between 0 and 1.	rand(2) ans = 0.9501 0.6068 0.2311 0.4860
rand(n,n)	As n×n matrix each values in the matrix is a random numbers between 0 and 1.	rand(3,2) Ans = 0.8913 0.0185 0.7621 0.8214 0.4565 0.4447

Gaussian Random Numbers:- (Normal Random Number)

Suppose that a random numbers Sequence represent temprelure messurement have some variation but tibically on

not equaly likely.

For ex. large changes accationaly accure cloud shadows and day to night.

Random numbers sequence that have some values that are more likely to accure other can often to be modeled with a Graussian Random variable (Normal Random variables).

`rand n(n)` Returns an nxn matrix. Each value in the matrix is a Graussian (or normal) random number with a mean of 0 and a variance of 1.

```
rand n(2)
ans =
-0.4326   0.1253
-1.6656   0.2877
```

`rand n(m,n)` Returns an mxn matrix, Each value in the matrix is a Grassian (or normal) random number with a mean of 0 and a variance of 1.

```
rand n(3,2)
ans =
-1.1465   0.0376
 1.1909   0.3273
 1.1892   0.1746
```

Defining Matrices:

We know that the Matrix can be defined by typing in a list of numbers enclosed in square brakat.

The numbers can be seperated by spaces or commass at the users discretion. New rows are indicated with a semicolon.

Example: A = [3, 5];

B = [1.5, 3.1]; or B = [1.5 3.1];

C = [-1, 0, 0; 1, 1, 0; 0, 0, 2];

A matrix can also defined by seperate line, as in the following set of MATLAB command.

```
C = [-1, 0, 0;
      1, 1, 0;
      0, 0, 2];
```

Colorem operator:

The colon operator is a powerful operator for defining a new matrices and modifing existing matrices. An evenly Spaced matrix can be defined with the colon operators the H = 1:8

will return

```
H =
    1  2  3  4  5  6  7  8
```

Ex:-

time = 0.0 : 0.5 : 2.0

will return

time =

0.0 0.5 1.0 1.5 2.0

Ex:-

We defined

M = [1 2 3 4 5 ; 2 3 4 5 6 ; 3 4 5 6 7] ;

MATLAB command

X = M (: , 1)

return,

X =
1
2
3

Also,

Y = M (: , 4)

will return,

Y =
4
5
6

Similarly

Z = Y (1 , :)

return,

Z = 1 2 3 4 5

Also,

N = M (2 : 3 , :)

return

$$W = \begin{matrix} 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \end{matrix}$$

and $W = M(2:3, 4:5)$

return

$$W = \begin{matrix} 5 & 6 \\ 6 & 7 \end{matrix}$$

In MATLAB it is valid to have a matrix that is empty.

for example the following statement to generate and empty matrix.

```
a = [];
b = 4:-1:5;
```

Finally M(:)

Here M is one column matrix.

Example:

$$M = \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \end{matrix}$$

M = (2,3) M(8)

Ans = 4 Ans = 4

or

Computational limitation:-

The variable stroage in a computer can assume the little range of values, the most computers the range from 10^{-308} to 10^{+308} and which should be enough to accomedate most computation. It is possible for the result of an expression to be outside of this range, suppose that we execute the following command. Example:1

```
x = 2.5e280;
y = 1.0e200;
z = x*y
```

MATLAB responds with

Z = inf

Example: 2

```
x = 2.5e-200
y = 1.0e200
z = x/y
```

MATLAB responds with

Z = 0

Example: 3

```
z = y/0
```

will returns

Z = inf

Special values and functions:-

MATLAB include a number of predefind constant, special values and

matrices are available to the programs. Most of the special values and the special matrices are generated by MATLAB by using functions. the following feenctions are,

Special Characters

eps - Smallest difference recognized

i - imaginary numbers

Inf - infinity

j - imaginary numbers

Nan - Not a number

Pi - Mathematical constant, π

Commands and functions

abs - computes the absolute value

asin - computes the inverse sine (arcsine)

ceil - rounds on the nearest integer toward positive infinity.

cos - computes the cosine

cumprod - computes a cumulative product of the values in an array.

cumsum - computes a cumulative sum of the value in an array.

erf - calculates the error function

exp - computes the value of e^x

fix - rounds to the nearest integer toward zero

floor - rounds to the nearest integer toward minus infinity.

help - opens the help function.

length - determines the largest dimension of an array

log - computes the natural log

$\log_{10}$ - computes the log base 10

$\log_2$ - computes the log base 2.

max - finds the maximum value in an array, and determines which element stores the maximum value.

Mean - computes the average of the elements in an array.

Median - finds the median of the elements in an array.

min - finds the minimum in an array, and determines which the minimum value.

Prod - multiples the value in an array.

rand - calculates evenly distributed random numbers.

randn - calculates normally distributed (Gaussian) random numbers.

rem - calculates the remainder in a division problem.

round - rounds of the nearest integer

Sign - determines the sign (positive or negative)

Sin - computes the sine

Sin h - compides the hyperbolic sine

Size - determines the number of rows and columns in an array.

Sort - sorts the elements of a vector into ascending order.

Unit - II

Plotting :-

Two dimensional plots :-

The most common plot used by engineers is the x-y plot the data, we plot usualy read from a data file. The data that the

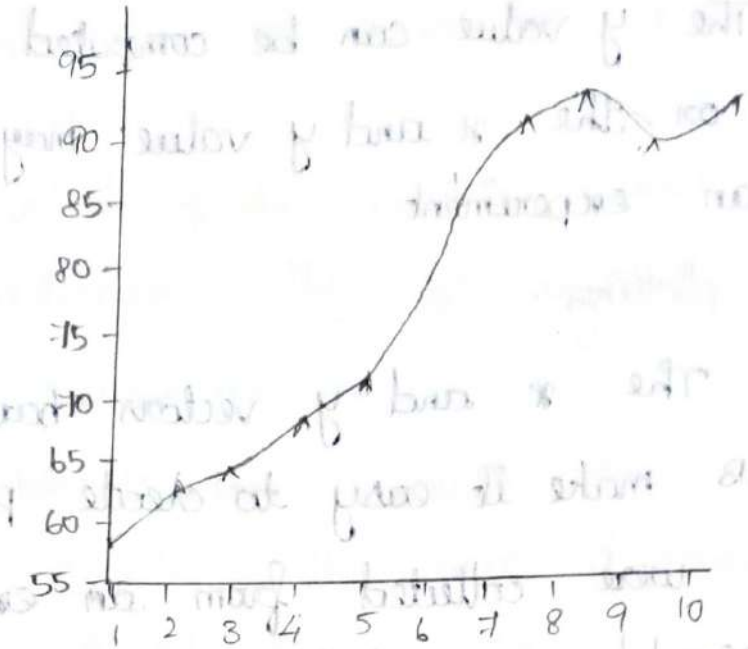
The y value can be computed as a function of x or the x and y value may be messured in an experiment.

Basic plotting :-

The x and y vectors have been defined MATLAB make it easy to crecde plots. The data were collected from an experiment with a remotely controled model car. The experiment was repeated ten times, we have messured the distance that the car traveled for each trial as shows below. Assume that the trial values are stored in a vector called x. An the distance values are storted in a vector called y.

Trial	Distance
1	58.5
2	63.8
3	64.2
4	67.3
5	71.5
6	58.3
7	90.1
8	90.6
9	89.5
10	90.4

```
x = [1:10];
y = [58.5, 63.8, 64.2, 67.3, 71.5, 58.3, 90.1, 90.6, 89.5,
90.4]
```

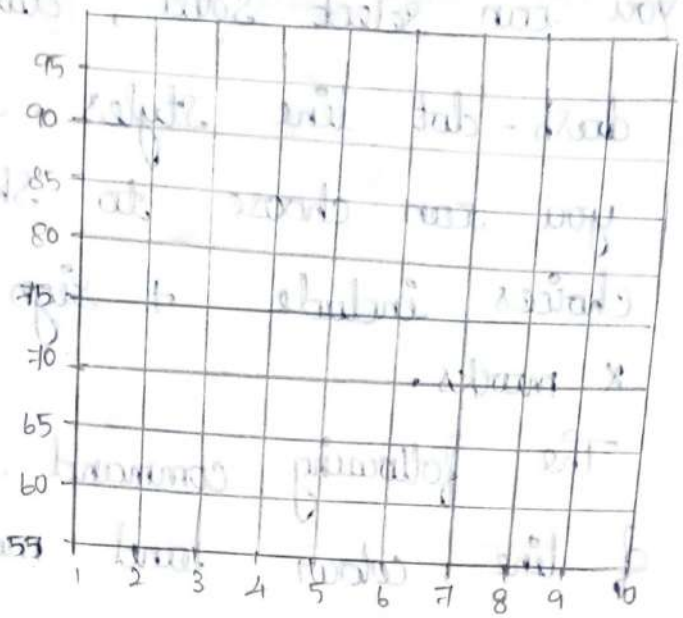


To plot these points, we use the plot with x and y as argument plot (x, y)

A graphics window automatically open with MATLAB the resulting plot is stored in the figure 1.

The following commands added a title, x and y axis labels and a background grid.

```
plot (x, y)
title ('Laboratory Experiment 1')
x label ('Trial')
Y label ('Distance')
grid on
```



Line, colour, and Mark Style:

The command plot (x, y) generate and xy plot the connect the points represent by the vectors x and y with straight line segments.

Graphs that appear to be curve are actually created by using a large number of points but still connecting them with very short straight line segment in the default mode, MATLAB uses a blue line for the first set of data plotted. And the changes the colour for subsequences set of data.

You can change the appearance of your plots by selecting user defined line strain And line colours and by us choosing to show the data points on the graph with user specify mark style sign by helping

you can select solid, dashed, dotted and dash-dot line styles and line colours and you can choose to show the points. The choices include + sign, stars, circle and x marks.

The following command illustate the use of line, colour and mark style.

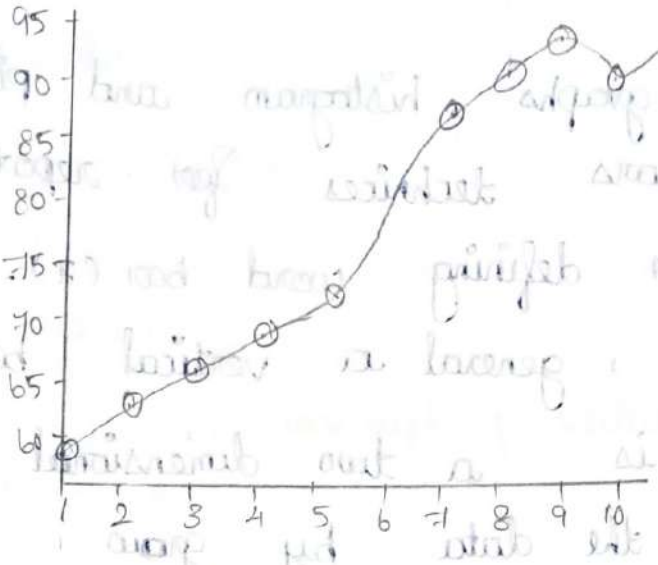
```
x = [1:10];
y = [58.5, 63.8, ....., 90.4];
plot (x, y, '-:ok')
```

Table: Line, Mark, color

Line type	Indicator	Point type	Indicator	Color	Indicator
solid	-	Point	.	blue	b
dotted	:	circle	o	green	g
dash-dot	-.	x-mark	x	red	r
dashed	--	plus	+	cyan	c
		star	*	magenta	m
		square	s	yellow	y
		diamond	d	black	k

Point type	Indicator
triangle down	v
triangle up	^
triangle left	<
triangle right	>
Pentagram	p
hexagram	h

Adjusting the line, mark and color style



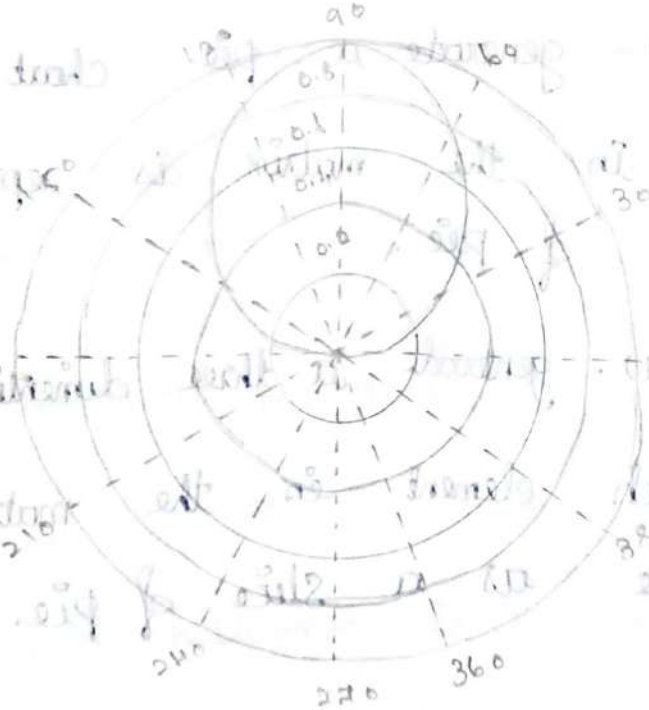
Other types of two dimentional plot:-

Polar plots:-

MATLAB provide plotting capability with polar co-ordinates. polar (theta, rho).

Generate the polar plot of angle theta and radial distance rho generate the plot in diagram.

title ('The sine function plotted in polar coordinates is a circle.')



Bar Graphs and pie charts:-

Bar graphs, histogram and pie charts and populars techniques for reporting data. The user defining word bar(x). Then x is a vector, general a vertical bar graphs. Then x is a two dimensional matrix bar graphs the data by grow

Then x is a vector, barh generate a horizondal bar graph. Then x is a two dimentional matrix barh groups data by rows bar3(x) - generate a three dimentional bar chart.

barh3(x) - generate a three dimentional horizondal bar chart.

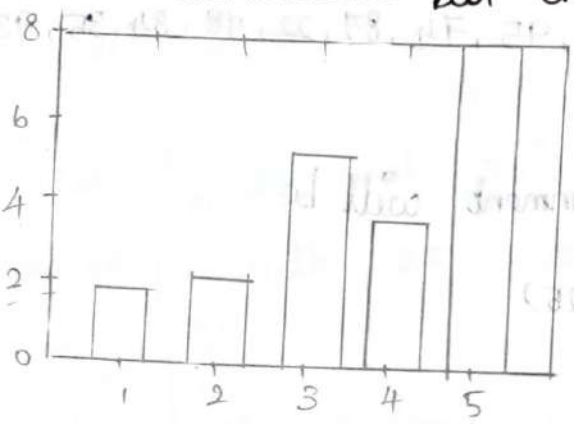
Pie(x) - generate a pie chart each element in the matrix is representate as a slice of pie.

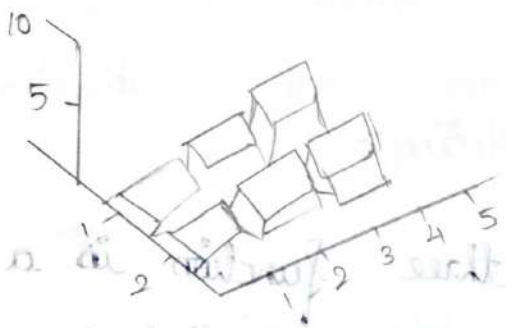
Pie3(x) - generate a three dimentional pie chart each element in the matrix is representate as a slice of pie.

Example:

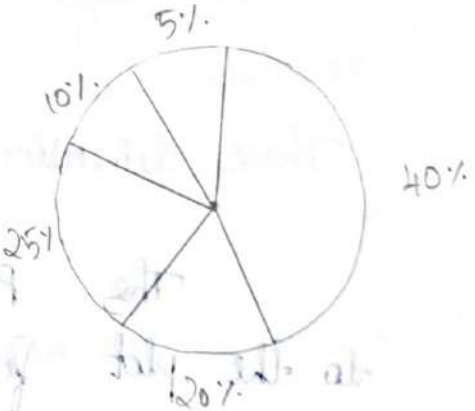
```
clear, clc
X = [1,2,5,4,8];
Y = [X; 1:5];
subplot (2,2,1)
bar(x), title ('A baragraph of vector x')
subplot (2,2,2)
bar(y), title ('A baragraph of matrix Y')
```

Two dimensional bar charts





Three dimensional bar charts



Pie chart

Histogram:

A histogram is a special type of graph particularly relevent to

Statistic. A histogram is a part sharing the distributing of a set of values in MATLAB the histogram computers the number of value falling in 10 bins that are equally space between the maximum (or) minimum values.

Example:

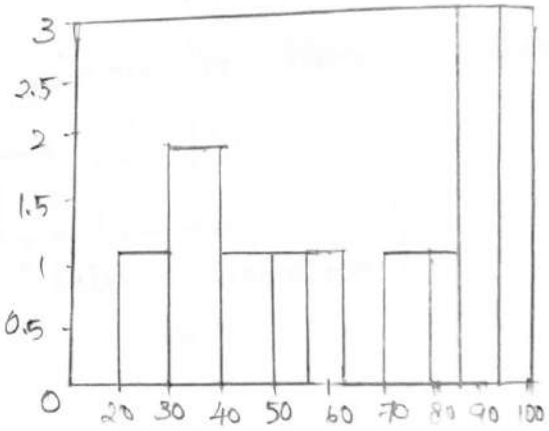
The score representate in a histogram

X = [100, 95, 74, 87, 22, 78, 34, 35, 93, 88, 86, 42, 55, 48]

hist(x)

MATLAB comment will be

hist (x, 25)



Three dimensional plotting:-

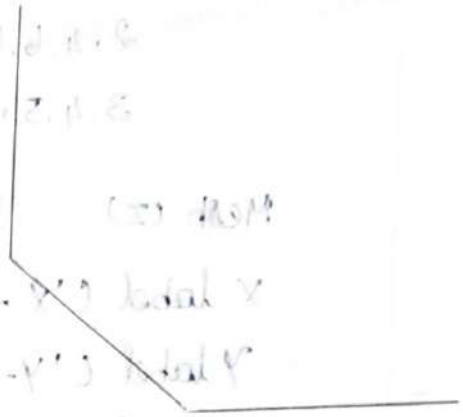
The plot three function is a similar to the plot function except that is accept data in three dimensional instead of just provide x and y vectors, the uses must also provide a z vector.

These ordered "Triples" are the plotted in three spaces and connected

with straight line.

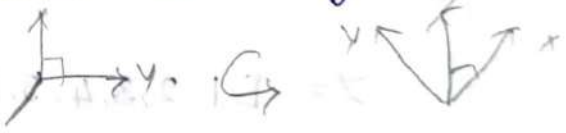
For example,

```
clear, clc
x = linspace(0,10*pi,100);
Y = cos(x);
Z = sin(x);
plot3(x,Y,Z)
grid
x label('angle')
Y label('cos(x)')
Z label('sin(x)')
title('A Spring')
```



The title, label and grid are added in the usual way, with the addition of z label for the z-axis.

The coordinate system used with plot3 is oriented using the right handed coordinate system familiar to engineer.



Surfaces plots:

Surface plot allow to represent data as a surface we will be engineering with mesh plots and with surf plots.

Mesh plots:

The matrix represent the z value

in the plot. The x and y value are based on the matrix dimensional. For example,

```
Z = [ 1,2,3,4,5,6,7,8,9,10;
      2,4,6,8,10,12,14,16,18,20;
      3,4,5,6,7,8,9,10,11,12 ]

Mesh (Z)
X label ('X - axis')
Y label ('Y - axis')
Z label ('z - axis')
```

The graph is a mesh created by connecting a point defined in Z into the rectilinear grid.

The mesh function also we used with three argument mesh(X, Y, Z) when each is a two dimensional matrix.

```
Z = [1,2,3,4,5,6,7,8,9,10;
     2,4,6,8,10,12,14,16,18,20;
     3,4,5,6,7,8,10,11,12 ]

X = line space (1,50,10)
Y = line space (500,1000,3)
```

I Student Edition of MATLAB

MATLAB Windows

Command Window

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Workspace Window

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Scalar operations

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Trigonometric function

Variance and Standard Deviation

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III Linear Interpolation

Linear Regression

Polynomial Regression